Session 1

Multiplication Clusters

Materials

- Interlocking cubes (at least 60 per student)
- One-centimeter graph paper (1–2 per pair)
- Student Sheet 10 (1 per student, homework)

What Happens

Students solve groups ("clusters") of related multiplication problems. They look for patterns and relationships within each cluster that can help them solve the last problem and then write about their problem-solving strategies. Their work focuses on:

- using multiplication relationships
- breaking down large problems

Activity

Solving Cluster Problems

Remind students what cluster problems are (you may want to read the **Teacher Note** Cluster Problems on p. 47).

For the next few days we'll be thinking about good strategies for figuring out hard multiplication problems. We'll be looking at groups of multiplication problems that are related to one another. These groups of problems are called Cluster Problems.

When we did Cluster Problems before, you used the first two problems to help you solve the third problem. These clusters sometimes have four or five problems in each cluster. Again, you'll use the earlier problems to help you solve the last problem in the cluster. You can use the interlocking cubes or the graph paper to make arrays to help you solve these problems. But try to solve the last problem by thinking about the other problems in the cluster. You can add problems to the cluster that help you solve the final problem better.

Present the following cluster:

4×5
2×15
4×10
4×15

Students talk in small groups about how to solve this first cluster. They can work on the first three problems in any order, then decide how to use one or more of those problems to solve 4×15. When they have finished, have volunteers show how they did the problems to the whole class.

Present one more cluster to the class.

2×8
6×8
10×8
12×8

Students complete the problems and compare strategies within their groups. As the students are working, circulate among the groups and note the strategies students are using.

Sharing Strategies Bring the whole class together briefly to share strategies they have figured out for themselves or learned from one another. Possible questions you might ask to help focus their thinking are:

> **Did you learn something useful from someone else in your group? Tell us about it.**
> **Which problems did you use to solve the last problem?**
> **Did anyone solve the problem in a different way?**
> **Did anyone add a different problem to the cluster to help solve 12×8?**

Activity

Writing About Your Strategies

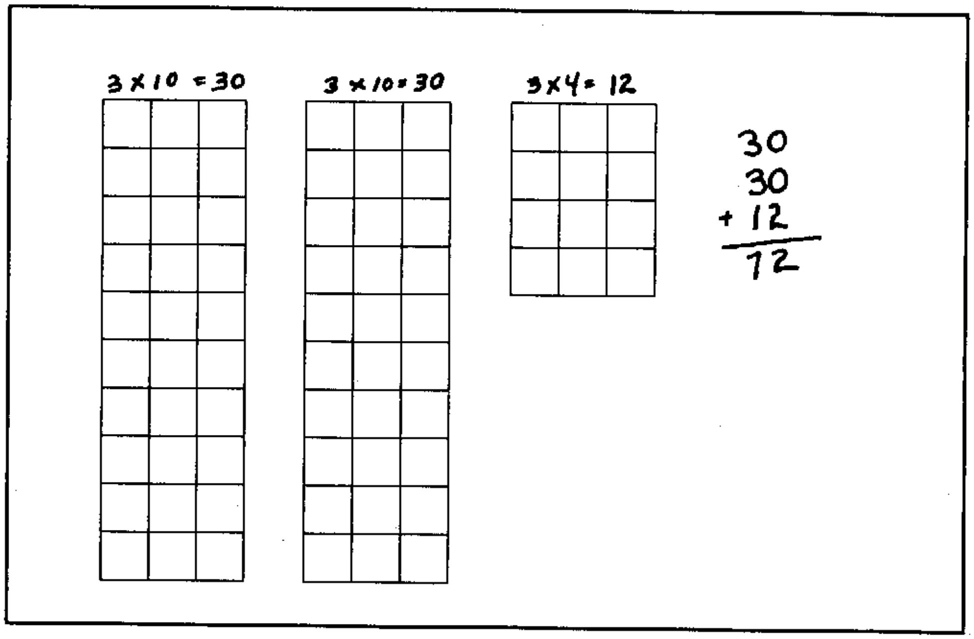
Show a third set of problems. Suggest to students that they solve mentally any of the first three problems they can and write down the answers, then use those answers to solve the rest of the problems, including the last problem in the cluster. As always, they should write down any different problems they use to help them solve 3×24.

3×10
3×20
3×4
3×24

This time, students work alone to solve the problems. They write about how they solved them, then create a set of arrays to show their thinking.

❖ **Tip for the Linguistically Diverse Classroom** Have limited-English-proficient students demonstrate their thought processes by illustrating their strategies. Omit instructions that direct students to write about their approach.

One way to solve 3 × 24 is with centimeter graph paper:



The **Dialogue Box,** Ways to Solve 3 × 24 (p. 48), gives some examples of student strategies for solving this multiplication cluster.

Session 1 Follow-Up

Homework

Another Set of Related Problems Assign the set of cluster problems on Student Sheet 10, Another Set of Related Problems.

Students find the solution to each problem and write about how they solved the last problem. Provide time in the next session for students to compare their solutions and strategies with classmates. Encourage students to invent their own set of cluster problems they can share with classmates.

❖ **Tip for the Linguistically Diverse Classroom** Have limited-English-proficient students demonstrate their solutions by illustrating their strategies as shown in the example on this page. Omit the instructions that direct them to write about their approach and to invent their own set of cluster problems.

Cluster Problems

Teacher Note

Cluster problems are sets of problems that help students think about using what they know to solve harder problems. For example, what do you know that would help you solve 12 × 3? If you know that 3 × 3 = 9, you might double that solution to get the solution to 6 × 3. Now that you know that 6 × 3 is 18, you can double that to get 12 × 3. Or, you might start with 10 × 3. If you know that 10 × 3 = 30, then you can start with 30 and add two more 3's to get 36. As students work with clusters, they learn to think about all the number relationships they know that might help them solve the problem they are working on.

The cluster problems used in this unit are designed to help students make sense of multiplying two-digit numbers. They build an understanding of the process by pulling apart multiplication problems into manageable subproblems, solving each of the smaller problems, then putting the parts back together. This process is based on an important characteristic of multiplication called the *distributive property.* You may have learned the name of this property in your own schooling without ever quite understanding what it is. In this unit, we don't teach students the name of the property, but it *is* a core idea of the unit. Here is an example: 9 × 23 can be thought of as (9 × 10) + (9 × 10) + (9 × 3). In this example, 23 is pulled apart into 10 + 10 + 3, and *each part* must be multiplied by 9 in order to give the solution to 9 × 23. The number that we split up doesn't have to be split up into 10's and 1's. For instance, 8 × 12 could be taken apart into (4 × 12) + (4 × 12) or into (8 × 6) + (8 × 6). In each case, one of the numbers is split up into parts, and each part must be multiplied by the other number in order to maintain equivalence to the original expression–8 × 12 = (4 × 12) + (4 × 12) or 8 × 12 = (8 × 6) + (8 × 6).

SET C

2 × 5 =10
3 × 5 =15
10 × 5 =50
30 × 5 =150
32 × 5 =160

on the last one I knew that 30×5=150 So I did
5×2=10
30×5=150
150+10=160

SET D

5 × 7 =35
10 × 7 =70
4 × 25 =100
20 × 7 =140
25 × 7 =175

It was easier to use 25's, so I used 4×25=100, then you just need 3 more 25's, 125, 150, 175.

As students solve the first few problems in each cluster, they continue to become familiar with the single-digit multiplication pairs. Students will begin to say "I just knew it" for some of these pairs, as they become part of their known repertoire of multiplication combinations. They will also make use of multiplying by 10 and by multiples of 10, another essential tool in solving harder multiplication problems (see the **Teacher Note,** Multiplying by Multiples of 10, p. 54).

Cluster problems are intended to help students learn how to look at a problem and build a strategy to solve it based on the number relationships they know. At first, students work on clusters of problems that are provided for them. Later in this unit, students begin to create their own clusters of problems. (In later units of *Investigations* in both grades 4 and 5, students spend more time creating their own clusters of problems as well as using a variety of given problems to solve harder computation.) Throughout their work on cluster problems in this unit, encourage students to add to the clusters any problems that they use to solve the final problem in the cluster.

DIALOGUE BOX

Ways to Solve 3 × 24

The class has been working on the following cluster problem in the activity Writing About Your Strategies (p. 45). They have written about how they have solved the cluster and are sharing their strategies for thinking about 3 × 24 in a whole-class discussion.

3 × 10
3 × 20
3 × 4
3 × 24

Kumiko: The way I did these was to count by each number. For 3 × 10, I counted by 10's. For 3 × 20, I counted by 20's. I just knew 3 × 4 was equal to 12. Then for 3 × 24, I added three 24's.

Rafael: I knew that 3 × 20 was 60 from the second problem. I knew that 3 × 4 was equal to 12, then I added the two together and got 72.

Ahmad: The way I figured out 3 × 24 was I know 24 plus 24 is equal to 48, so I added 2 from the next 24 and that made 50. Then I had to add the 22 that was left to the 50, and this gave me 72.

Shoshana: At first I didn't know how to do 3 × 24, but I knew that 3 × 20 = 60. So I tried to add 4 to the 20 and that made 24, *[Shoshana wrote 3 × 24 = 60]*. Then I added 4 to the 60 and got 3 × 24 = 64. But now I can see I made a mistake because I should have added three 4's to 60, not just one.

Vanessa: Mine was sort of like Ahmad's way. I just knew that 24 plus 24 is equal to 48, then I added 24 more and got 72.

Joey: I counted by 24's: 24, 48, 72!

Nick: I took 60 from 3 × 20, then I added 4 more 3's and got 72.